Global Indeterminacy in a Model with Public Health Spending

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Abstract

This paper investigates the possibility of global indeterminacy in a simple growth model with a non-separable utility function involving consumption and public health services provided by government through public health spending. According to the assumption of non-separability in utility, we show that long-run equilibria may exhibit global indeterminacy if the intertemporal elasticity of substitution is sufficiently large, in conjunction with the effects of public health. The condition for such indeterminacy is independent of the production sector. Numerical computations under reasonable parameter constellations support the theoretical result.

Keywords: Global indeterminacy; Public health spending; Non-separable utility function.
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1　Introduction

In recent years, a large body of research has investigated the existence of indeterminate equilibria in various types of models. The indeterminacy of equilibria typically indicates that there exist multiple transitional growth paths converging to a same (single) long-run equilibrium. This means that the perfect foresight equilibrium exhibits *local* indeterminacy.[1)]

The central issue in research on indeterminacy is that it provides an explanation for why economies with similar fundamentals experience large variations in growth performance. Thus, from the viewpoint of development macroeconomics, the concept of *global* indeterminacy which implies similar economies arrive at different long-run equilibria also deserves careful attention.[2)] In a typical situation, the model tends to have dual long-run growth equilibria. In such case, high and low growth rate equilibria coexist.

Here, we construct a simple growth model with public health services provided by government spending through income taxation, and examine its long-run properties. As a critical source of multiplicity, standard analyses on indeterminacy often assume (socially) increasing returns to scale for production technologies; however, our production function satisfies constant returns.[3)]

One of the notable features of the model is that it employs a non-separable utility function between consumption and the level of pubic health services. A recent paper by Fernández et al. (2004) provides an analysis under non-separable utility that includes private and public consumption and leisure, but they study only the local dynamics.[4)] Our concern is different from theirs in that we aim at global properties.

In light of the fact that governments have a responsibility to maintain public health in general, in this paper we assume that private agents maximize their own utility, taking the level of public health (that affects utility levels) as exogenously given. This is an adequate assumption given the observation of health care policies in many countries. In addition, public health is assumed to boost labor productivity in the production of goods like a labor augmenting technological progress.

The purpose of this paper is to clarify the possibility of global indeterminacy (multiple

1) Examples of recent typical contributions on local indeterminacy include Alonso-Carrera and Freire-Serén (2004), Benhabib and Perli (1994) and García-Belenguer (2007).

2) Although unlike our mechanism, recent contributions on global indeterminacy include Palivos et al. (2003), Park and Philippopoulos (2004) and Pérez and Ruiz (2007).

3) To generate (local) indeterminacy, for example, Benhabib and Perli (1994) and Boldrin and Rustichini (1994) rely on increasing returns with respect to production.

4) Bennett and Farmer (2000) have also introduced non-separable utility to examine the existence of local indeterminacy. Cazzavillan (1996) and Raurich (2003) show the emergence of local indeterminacy using a model that introduces public spending (or publicly provided goods) into the utility and production functions.

equilibria). What is important is whether one can establish the existence of dual long-run equilibria even when using an extremely simple model with public health spending. The principal results obtained in this paper are as follows. The emergence of global indeterminacy depends on the external effects of public health on the agent's preference and the size of the intertemporal elasticity of substitution. In particular, we show that there exist two balanced growth rate equilibria for sufficiently large values of intertemporal elasticity. This implies that countries with similar economic fundamentals may arrive at different equilibria in the long-run. We also confirm such multiplicity by running numerical computations under reasonable parameter constellations.

2 The Model

This section presents a simple growth model with public health spending in a decentralized economy setting. The key factor of the model is public health services provided by the government sector. The provision of public health services produces effects in two directions: a good public health environment increases labor productivity in the production of goods (e.g. such an environment will be helpful in extending healthy working hours through prevention of illness) and an improvement in economy-wide average health level directly enhances a representative agent's welfare. The latter will be explained later.

We assume that the production function for a representative firm is expressed as a standard Cobb–Douglas technology:

$$Y(t) = K(t)^{\alpha}[A(t)L(t)]^{1-\alpha}, \quad \alpha \in (0,1), \tag{1}$$

where Y, K, A and L are, respectively, aggregate output, physical capital, productivity index and labor force, and α is the share of physical capital in the production process.[5] As noted above, productivity is linked directly to the level of public health services, $A = H$. We also assume no population growth (i.e. $\dot{L}/L = 0$). Accordingly, from Eq. (1), we obtain $Y = K^{\alpha}(HL)^{1-\alpha}$. Therefore, the health factor has the characteristic of augmenting labor.

The representative firm's profit π is given by

$$\pi \equiv Y - rK - wHL = K^{\alpha}(HL)^{1-\alpha} - rK - wHL, \tag{2}$$

where r and w denote the rental price on physical capital and the wage rate, respectively. Standard profit maximization based on Eq. (2) gives

$$r = \alpha\frac{Y}{K}, \tag{3}$$

5) From now on, we will suppress the time argument t when not needed for clarity.

$$w = (1-\alpha)\frac{Y}{HL}. \tag{4}$$

Next, we need to explain government activities. We assume that the government maintains a balanced budget at each point in time. We obtain

$$G = \tau Y, \tag{5}$$

where G is government spending (for public health) and $\tau \in (0, 1)$ is the rate of income tax imposed on a representative household.

The household maximizes the following intertemporal utility function:

$$\int_0^{+\infty} \frac{(C\bar{H}^{\sigma})^{1-\theta}-1}{1-\theta}e^{-\rho t}\mathrm{d}t, \quad \sigma \in (0,1], \ \theta > 0, \ \theta \neq 1, \ \rho > 0, \tag{6}$$

where C is consumption and $\bar{H}$ is the economy-wide public health services exploited from the stock of public health capital. It has been recognized by, for example, the empirical findings of Viscusi and Evans (1990) that health status has an effect on utility. They find that the marginal utility of consumption (or income) increases with better health.[6] The idea of Eq. (6) stems from these findings. Moreover, σ is the weight of public health in utility, θ is the inverse of the intertemporal elasticity of substitution and ρ is the subjective discount rate. Note that $\bar{H}$ is an exogenous variable for the agent, thus the agent cannot choose that level.[7] Following Turnovsky (1996), $\bar{H}$ is defined by $\bar{H} \equiv H^{\beta}(H/Y)^{1-\beta}$. In this relation, the negative effect of Y to the level of public health services $\bar{H}$ captures what is called congestion phenomena associated with the character of public good. The weighting parameter $\beta \in [0, 1]$ corresponds to the degree of congestion. Here, we assume that public health is a pure (non-rivalling and non-excludable) public good (i.e. $\beta = 1$). This means $\bar{H} = H$. Applying this relation to Eq. (6), we obtain

$$\int_0^{+\infty} \frac{(CH^{\sigma})^{1-\theta}-1}{1-\theta}e^{-\rho t}\mathrm{d}t. \tag{7}$$

By using Eqs. (3)−(5), the household's flow budget constraint is given by

$$\dot{K} = (1-\tau)(rK + wHL) - C, \quad K(0) = K_0 > 0. \tag{8}$$

In Eq. (8) we omit physical capital depreciation. To solve the agent's dynamic optimization

6) For more a comprehensive discussion, see for example Smith (1999).

7) For this reason, a joint concavity assumption imposed on C and $\bar{H}$ (i.e. $\theta > \sigma/1+\sigma$) is not needed in the present case.

problem, let us define the current-value Hamiltonian $\mathcal{H}$. In the following, for analytical simplicity, we set the total labor force as normalized to unity ($L = 1$) such that all variables become per capita amounts. From Eqs. (7) and (8)

$$\mathcal{H} \equiv \frac{(CH^{\sigma})^{1-\theta} - 1}{1-\theta} + \lambda[(1-\tau)(rK + wH) - C],$$

where λ is the co-state variable associated with Eq. (8). The necessary optimality conditions are given by the following:

$$\lambda = (CH^{\sigma})^{-\theta} H^{\sigma}, \tag{9}$$

$$\dot{\lambda} = -\lambda(1-\tau)r + \lambda\rho. \tag{10}$$

The necessary conditions of Eqs. (9) and (10) are also sufficient when the transversality condition $\lim_{t\to+\infty} \lambda(t)K(t)e^{-\rho t} = 0$ holds.

Log-differentiation of Eq. (9) with respect to time gives

$$\frac{\dot{\lambda}}{\lambda} = -\theta\frac{\dot{C}}{C} + \sigma(1-\theta)\frac{\dot{H}}{H}. \tag{11}$$

Next, using the production function $Y = K^{\alpha}H^{1-\alpha}$ and Eq. (3), we obtain $r = \alpha(K/H)^{\alpha-1}$. Substitution of this relation to Eq. (10) yields

$$\frac{\dot{\lambda}}{\lambda} = -\alpha(1-\tau)\left(\frac{K}{H}\right)^{\alpha-1} + \rho. \tag{12}$$

From Eqs. (11) and (12), we get

$$-\theta\frac{\dot{C}}{C} + \sigma(1-\theta)\frac{\dot{H}}{H} = -\alpha(1-\tau)\left(\frac{K}{H}\right)^{\alpha-1} + \rho. \tag{13}$$

To complete the model, we need to explain the evolution of public health. As noted earlier, the stock of public health, H, evolves through governmental health spending, G. Following Capolupo (2000), the dynamic equation for public health capital is

$$\dot{H} = \delta G = \delta\tau Y = \delta\tau K^{\alpha}H^{1-\alpha}, \quad \delta > 0, \tag{14}$$

where δ represents the technological effciency parameter of public health creation.[8)] Combining Eqs. (13) and (14) and rearranging gives

$$\theta\frac{\dot{C}}{C} - \delta\sigma\tau(1-\theta)\left(\frac{K}{H}\right)^{\alpha} = \alpha(1-\tau)\left(\frac{K}{H}\right)^{\alpha-1} - \rho. \tag{15}$$

2.1　Dynamic System and Definition of Global Indeterminacy

We summarize the dynamic system of the model. First, we can obtain from Eqs. (8), (14) and (15) the following three differential equations.[9)]

$$\frac{\dot{K}}{K} = (1-\tau)\left(\frac{K}{H}\right)^{\alpha-1} - \frac{C}{K}, \tag{16}$$

$$\frac{\dot{H}}{H} = \delta\tau\left(\frac{K}{H}\right)^{\alpha}, \tag{17}$$

$$\frac{\dot{C}}{C} = \frac{1}{\theta}\left(\delta\sigma\tau(1-\theta)\left(\frac{K}{H}\right)^{\alpha} + \alpha(1-\tau)\left(\frac{K}{H}\right)^{\alpha-1} - \rho\right). \tag{18}$$

In view of tractability, the present three dimensional system can be converted to a new system:

$$\frac{\dot{X}}{X} = \frac{\delta\sigma\tau(1-\theta)Z^{\alpha} + \alpha(1-\tau)Z^{\alpha-1} - \rho}{\theta} - (1-\tau)Z^{\alpha-1} + X, \tag{19}$$

$$\frac{\dot{Z}}{Z} = (1-\tau)Z^{\alpha-1} - X - \delta\tau Z^{\alpha}, \tag{20}$$

where $X \equiv C/K$ and $Z \equiv K/H$. In consequence, these equations characterize the dynamics of the model.

At this moment, we should define the situation of global indeterminacy in this paper. Based on Palivos et al. (2003), the following definition is introduced with slight modification.

Definition (Palivos et al., 2003, p. 91, Definition 2) *Under a given predetermined state variable* $Z(0) \equiv K(0)/H(0)$, *if there exist at least two possible initial conditions* $X_i(0) \in \mathbb{R}_{++}$, $i = 1, 2$ *such that*

8) Capolupo (2000) explores a Barro (1990)-type endogenous growth model with human capital in which the accumulation of human capital depends only on governmental education spending. In that case, one interpretation of the formulation is that human capital is accumulated through compulsory education. Therefore, as far as the evolution of public health is concerned, our model can be seen as the application of Capolupo (2000) to a public health setting.

9) Applying $r = (K/H)^{\alpha-1}$ and $w = (1-\alpha)(K/H)^{\alpha}$ to Eq. (8), we have Eq. (16).

these $X(X_i(0), t)$ converge to a steady-state equilibrium, we call the situation globally indeterminate.

As is obvious, the initial value of the jump variables $C(0)$ and therefore $X(0) \equiv C(0)/K(0)$ are not predetermined. On the basis of the definition, if there exist multiple equilibria in this model, that ensures the emergence of global indeterminacy independent of the local stability properties of these equilibria (Palivos et al., 2003). In the following section, we will investigate equilibrium properties in consideration of the possibility of global indeterminacy.

2.2 Balanced Growth Path Solution

In a balanced growth path (BGP), by definition, all endogenous variables C, K, H and Y grow at the same rate on the path. If denoting g as the common growth rate, $g \equiv g_C = g_K = g_H = g_Y$ holds.[10] Applying $g = g_H$ to Eq. (14) yields $K/H = (g/\delta\tau)^{1/\alpha}$. Substituting this into Eq. (15) together with $g = g_C$ gives

$$\theta g - \sigma(1-\theta)g = \alpha(1-\tau)\left(\frac{g}{\delta\tau}\right)^{\frac{\alpha-1}{\alpha}} - \rho. \tag{21}$$

As a consequence, the solution g^* to Eq. (21) corresponds to the balanced growth rate for the decentralized economy.

2.3 Equilibrium Properties and Global Indeterminacy

To investigate the equilibrium properties of the model, it would be convenient to rewrite Eq. (21) as

$$\underbrace{\alpha(1-\tau)\left(\frac{g}{\delta\tau}\right)^{\frac{\alpha-1}{\alpha}}}_{\Gamma} = \underbrace{[\theta-\sigma(1-\theta)]g+\rho}_{\Psi}. \tag{22}$$

Let the left-hand side (LHS) and the right-hand side (RHS) of Eq. (22) be denoted Γ and Ψ, respectively. Both Γ and Ψ are the function of g, namely, $\Gamma(g)$ and $\Psi(g)$. For each function, we characterize the shapes.

First, we find that $\Gamma(g)$ is a strictly decreasing and a strictly convex function of g, in view of functional properties $\lim_{g\to 0}\Gamma(g) = +\infty$, $\lim_{g\to 0}\Gamma'(g) = -\infty$, $\lim_{g\to +\infty}\Gamma(g) = 0$ and $\lim_{g\to +\infty}\Gamma'(g) = 0$.

On the other hand, $\Psi(g)$ is a simple linear function of g, but the slope varies depending on the size of θ. Specifically, there are three possibilities: first, when $\theta > \sigma/1+\sigma$, Ψ has a

10) The expression g_x denotes the growth rate of placeholder x.

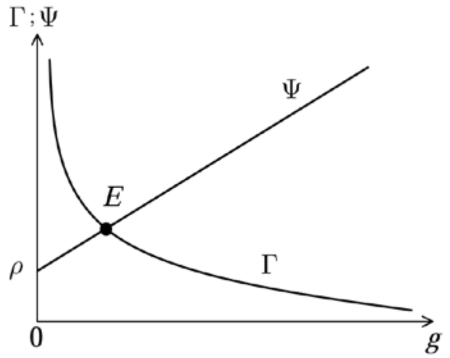

Figure 1: $\theta > \sigma / 1 + \sigma$

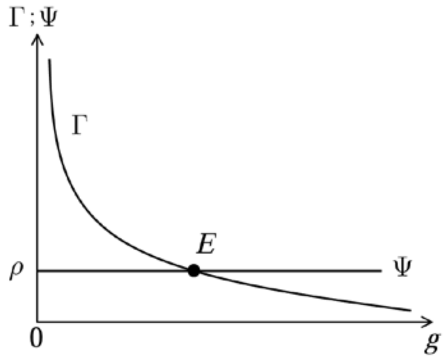

Figure 2: $\theta = \sigma / 1 + \sigma$

positive slope with intercept, $\rho > 0$, in the relevant quadrant; second, when $\theta = \sigma / 1 + \sigma$, Ψ comes to a horizontal line at ρ; and third, when $\theta < \sigma / 1 + \sigma$, Ψ has a negative slope with intercept ρ.

According to the shapes of Γ and Ψ, we can state the following two propositions.

Proposition 1 *When $\theta \geq \sigma / 1 + \sigma$, there exists a unique long-run equilibrium solution. This case is shown in Figure 1 (the case of $\theta > \sigma / 1 + \sigma$), and Figure 2 (the case of $\theta = \sigma / 1 + \sigma$).* □

Proposition 2 *There is a possibility that the global indeterminacy of long-run equilibria arises from a preference parameter condition of $\theta < \sigma / 1 + \sigma$.* □

To understand the above statement, we explore it further. That is, (i) when the LHS (Γ) is located below the RHS (Ψ) over the whole range of positive g, there is no solution; (ii) when the LHS is tangent to the RHS, that is, $\Gamma'(g^*) = \Psi'(g^*)$, there is a single solution; (iii) when the LHS is located above the RHS, there are two solutions.

The last case, (iii), implies the emergence of global indeterminacy; there exist two long-run growth rates that satisfy optimality criteria. The corresponding situation is presented in Figure 3. In this figure, an equilibrium E (E') represents a low (high) growth equilibrium. To confirm the possibility of our theoretical result for multiple growth paths, we resort to numerical computations. A numerical example is shown in Figure 4.[11] This example yields a dual BGP situation as seen in Figure 3. The following parameter values are applied: $\alpha = 0.3$, $\tau = 0.04$,

11) We used MATLAB 6.5.1 to run numerical computations.

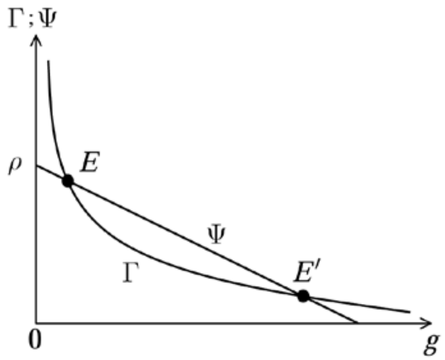

Figure 3: Global indeterminacy

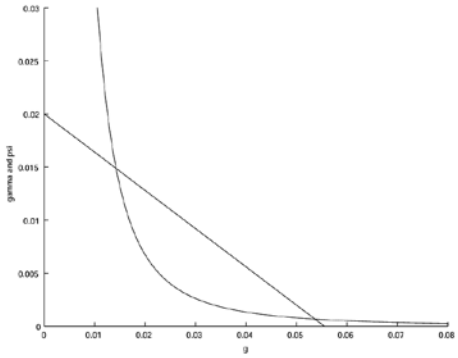

Figure 4: Numerical example

$\delta = 0.1$, $\sigma = 0.7$, $\theta = 0.2$, $\rho = 0.02$.[12] In the lower growth case (corresponding to E in Figure 3) the equilibrium growth rate is $g^* = 0.0142$, while in the higher growth case (corresponding to E') $g^* = 0.0537$.[13]

As explored above, theoretically and numerically, it is quite likely that our model exhibits dual balanced growth equilibria, and thus equilibrium is *globally indeterminate*. The important point in obtaining dual equilibria is the specification of preference structure. In particular, the crucial parameters for generating global indeterminacy are σ and θ within the agent's utility function. To put it more concretely, a positive value of σ, namely, the existence of the effects of public health services on welfare, is a necessary condition for global indeterminacy.[14] Additionally, if the intertemporal elasticity of substitution takes a sufficiently large value (i.e. θ is low enough) which satisfies the condition $\theta < \sigma/1+\sigma$, there is a possibility of global indeterminacy. It should also be added that public health services are an external factor for the optimal behavior of the representative household. As shown in several studies, some kinds of external effect have a deep connection with indeterminacy; therefore, the externality in preference explained above naturally affects the multiplicity of equilibria.

Accordingly, in the present analysis, both the agent's preference and the feature of health services as a public good have a significant role in generating global indeterminacy.

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12) From the formulation of Eq. (5), we can take τ as the share of public health expenditure to GDP ($\tau = G/Y$). According to *World Development Indicators 2007* (*CD-ROM*), for example, averaged public health expenditure (% of GDP) in 132 countries during 2000–2004 was 4.23%. Therefore, $\tau = 0.04$ is an appropriate value. Concerning σ and θ, we set these values in light of the condition of $\theta < \sigma/1+\sigma$.

13) We should present an additional numerical example. If updating σ and θ to 0.95 and 0.35 while keeping other parameters unchanged, the two long-run growth rates are 0.0137 and 0.0736, respectively.

14) In fact, global indeterminacy is never found in the case of $\sigma = 0$.

3　Concluding Comments

This paper has focused on only the long-run properties of general equilibrium at the BGPs, particularly on the problem of the emergence of global indeterminacy. Therefore, a further direction of this paper will be to investigate its local dynamics around the BGPs. This line of research is essential for a better understanding of the present study.

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