A Chamberlinian Agglomeration Model with External Economies of Scale*

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Abstract

We investigate the effects of reduction in trade cost on industrial location and welfare in an economy with external economies of scale. We develop a Chamberlinian agglomeration model with footloose capital, through which we demonstrate that a reduction in trade cost is likely to lead to industrial agglomeration, which makes a country with agglomeration better off and one without agglomeration either better or worse off, depending on the level of trade cost and the degree of external economies of scale.

Keywords: New economic geography; Agglomeration; Footloose capital, External economies of scale
JEL classifications: F12; F15; F21; R12

1 Introduction

In the last two decades, the effects of reduction in trade costs on industrial location have been the focus of study in the field of new economic geography. In the well-known, seminal work by Krugman (1991), the model, a so-called "core-periphery model," focuses on agglomeration in the industrial sector with trade cost, increasing returns to scale, and monopolistically competitive markets. In the core-periphery model, industrial workers are mobile between

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regions or countries, and the shifting of workers entails a shift in demand and, subsequently, a shift in firms. He shows that a reduction in trade cost causes drastic agglomeration at the threshold trade cost level − a "catastrophic result" − even in two countries that are initially symmetric. Subsequent extensions have provided analytically solvable frameworks (e.g., Forslid and Ottaviano, 2003; Pflüger, 2004) and have addressed economic welfare (e.g., Amiti, 2005; Chalot et al., 2006; Behrens, 2007; Pflüger and Südekum, 2008). In their model, a driving force for agglomeration is the movement of labor, and thus, they presume smooth labor movement between regions or countries.

In reality, however, there are some regions or countries where the movement of labor is not so smooth. For example, the United Nations (2011) reports that immigrants comprised 16.8%, 14.2%, and 9.5% of the population in Oceania, Northern America, and EU, respectively. In contrast, the share of immigrants in Asia is only 1.5%, which implies that the labor movement in Asia is not smooth. Nonetheless, we have witnessed some agglomeration in Asia, such as in Shanghai and in Guangzhou.

If the labor movement is not smooth, other factors must work as alternative driving forces for such agglomeration. A potential factor is the Marshallian externality, that is, external economies of scale. For example, Lu and Tao (2009) report strong evidence of the positive role of the external economies of scale in industrial agglomeration in China. Some empirical studies (e.g., Andretsch and Feldman, 1996; Rosenthal and Stranger, 2001) also support the notion that external economies of scale are crucial for industrial agglomeration.

In this paper, we propose an analytically solvable framework to investigate industrial location in regions or countries without smooth labor movement. We incorporate external economies of scale into the "footloose capital model" by Martin and Rogers (1995).[1] We consider the agricultural and manufacturing sectors in two countries that are initially symmetric. In the former sector, a homogeneous good is produced using only labor under a constant returns to scale technology, the market is perfectly competitive, and no trade costs are necessary. In the latter sector, differentiated goods are produced using labor and capital under an increasing returns to scale technology with external economies of scale. The market is monopolistically competitive, and iceberg trade costs must be incurred when the good is traded. Capital is mobile between the countries, and therefore, international distribution of capital and industrial location are endogenously determined in our model. Further, we focus on changes in welfare of these countries via industrial location.

1) Borck et al. (2012) also extend the "footloose-capital model" and examine industrial location and welfare under a subsidy game with intra-sectional and intersectional externalities. Their focus is mainly on the effect of the subsidy game on industrial location, which is quite different from our motivation.

Under this setup, we obtain the following results: With respect to industrial location, a reduction in trade cost is likely to cause agglomeration in the manufacturing sector (Proposition 1). In particular, if the trade cost becomes lower than the threshold level, drastic agglomeration occurs. That is, the "catastrophic" agglomeration, such as that in the core-periphery model, is obtained in our analytically solvable framework.

With respect to welfare, the country with agglomeration becomes unambiguously better off, whereas the country without agglomeration may become either better off or worse off (Proposition 2). This result implies that agglomeration may or may not be Pareto-improving. The above results are obtained depending on the level of initial trade cost and on the degree of external economies of scale.

Finally, we discuss how this paper relates to existing studies in the new economic geography. As we have already stated, we apply the "footloose capital model" in this paper and propose an analytically solvable framework to investigate industrial location and economic welfare in the regions or countries without smooth labor movement. Of the existing studies, Krugman and Venables (1995) is the one most closely related to ours. They consider an input–output linkage, which is a sort of Marshallian externality, as an agglomerative force. In their model, differentiated manufactured goods are used as intermediate inputs and consumption is denoted as final goods. Agglomeration creates forward and backward linkages in the economy. Under the model with this property, they show that agglomeration is not Pareto-improving. The result is based on a numerical simulation because of low tractability of the model. Although we also include the Marshallian externality − external economies of scale − as a driving force, we analytically demonstrate that agglomeration may or may not be Pareto-improving.

The rest of this paper is organized as follows. Section 2 proposes the basic model. Section 3 considers the short-run equilibrium and clarifies some characteristics of this economy. Section 4 derives the long-run equilibrium and examines the effects of a reduction in trade cost on industrial location and welfare. Section 5 briefly concludes the paper.

2 The Model

The economy is composed of two countries, home and foreign (denoted by an asterisk); two sectors, agriculture and manufacturing; and two factors of production, labor and capital.

In the agricultural sector, a homogeneous good is produced using only labor under a constant returns to scale technology. The market is perfectly competitive. No trade costs are necessary when the good is traded between countries. In what follows, we treat the agricultural good as the numeraire.

In the manufacturing sector, differentiated goods are produced using labor and capital under an increasing returns to scale technology. The market is monopolistically competitive. An iceberg trade cost must be incurred when the manufacturing good is traded. We assume that the iceberg trade cost from home to foreign is the same as that from foreign to home.

Labor is mobile between these sectors, but immobile between countries. Capital owners are also immobile, while capital is mobile between countries.[2] Capital moves to the country with the higher reward. All capital rewards are repatriated to the country that the capital owners inhabit.

The home and foreign countries are assumed to be symmetric on labor endowment and the number of capital owners, preferences and production technology. With respect to labor, each country is endowed with L units. With respect to capital, the overall economy is endowed with $\bar{K}$ units.

In the following, we provide some detail explanations of preferences, and production technology. From symmetry, we focus only on the home country.

2.1 Consumption

Consumers in the home country have a common Cobb-Douglas utility function:

$$U = C_M^{\mu} C_A^{1-\mu}, \quad 0 < \mu < 1,$$
$$C_M \equiv \left(\int_0^n c_i^{\frac{\sigma-1}{\sigma}} di + \int_0^{n^*} c_{i^*}^{\frac{\sigma-1}{\sigma}} di^* \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where C_M and C_A are the consumption of the aggregate of varieties of the manufacturing goods and that of the agricultural good, respectively; i and i^* are the indices of the manufacturing good produced in the home and foreign countries, respectively; n and n^* are the number of home and foreign varieties, respectively; μ is the share of expenditure on manufacturing aggregates, and $\sigma\,(>1)$ is the constant elasticity of substitution between manufacturing varieties.

Under utility function (1), we obtain the following demand functions for varieties i and i^* of the manufacturing good, c_i and c_{i^*}:

2) For analytical simplicity, we assume no trade costs must be incurred when capital moves between countries in our model. Yamamoto (2008) focuses on the role of trade costs for capital movement.

$$c_i = p_i^{-\sigma} G^{\sigma-1} \mu I, \tag{2}$$

$$c_{i^*} = (\tau p_{i^*})^{-\sigma} G^{\sigma-1} \mu I, \tag{3}$$

where p_i and p_{i^*} are the respective prices of varieties i and i^* set by the home and foreign firms, $\tau > 1$ is the iceberg trade cost,

$$G \equiv \left(\int_0^n p_i^{1-\sigma} di + \int_0^{n^*} (\tau p_{i^*})^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{4}$$

is a price index on the manufacturing aggregates, and I is consumers' income. We also obtain the demand function for agricultural goods, $C_A = (1-\mu)I$, from equation (1).

2.2 Production

In the agricultural sector, we assume that a unit of labor is necessary for a unit of production. On the other hand, labor and capital are assumed to be required for production in the manufacturing sector. The manufacturing sector is subject to external economies of scale, which is modeled by considering technology such that an increase in the number of firms reduces the requirements of both labor and capital. In our model, we consider technology such that labor is used for the marginal requirement, while capital is used for the fixed requirement. Let $B(n)$ and $F(n)$, respectively, be the marginal labor requirement and the fixed capital requirement to produce q_i. $B(n)$ and $F(n)$ are given by

$$B(n) \equiv \left(1 - \frac{1}{\sigma}\right) n^{-\beta}, \quad \beta > 0, \tag{5}$$

$$F(n) = n^{-\gamma}, \quad 0 < \gamma < 1, \tag{6}$$

where β and γ are parameters expressing external economies of scale on the marginal requirement and the fixed requirement, respectively. Note that an increase in the number of varieties, n, reduces $B(n)$ and $F(n)$.

3 Short-run Equilibrium

Before analyzing industrial location and economic welfare, let us consider the short-run equilibrium without international capital movement to clarify the working of the model, in particular, the mechanism to determine the rental rate. We focus on the situation where the amount of capital employed in each country is fixed at this moment. In the following analysis, we confine our attention to the situation where both countries produce agricultural goods and open their goods markets.[3)]

3.1 Derivation of the equilibrium

Let K and K^* be the amounts of capital employed in the home and foreign countries, respectively. Note that K and K^* do not correspond to the capital owned by the home and foreign countries. From equation (6), we obtain the equilibrium number of varieties in the short-run as

$$n = K^{\frac{1}{1-\gamma}} \quad \text{and} \quad n^* = K^{*\frac{1}{1-\gamma}}. \tag{7}$$

The agricultural good is the numeraire, so wage rate becomes unity in the equilibrium. Then, using equations (5) and (6), the profit for the firm producing variety i is organized as

$$\pi_i = p_i q_i - B(n) q_i - rF(n), \tag{8}$$

where q_i is the output for variety i, and w and r are the wage and rental rates, respectively, in the home country. From equation (8), the first-order condition for profit maximization is

$$p_i \left(1 - \frac{1}{\sigma}\right) - B(n) = 0. \tag{9}$$

Since free entry and exit are allowed, firms' profits are zero. Setting equation (8) to be zero, we obtain

$$rF(n) = \frac{1}{\sigma} p_i q_i. \tag{10}$$

Using equations (5), (7), and (9), we obtain

$$p_i = K^{-\frac{\beta}{1-\gamma}} \quad \text{and} \quad p_{i^*} = K^{*-\frac{\beta}{1-\gamma}}. \tag{11}$$

From equations (6), (7), and (11), the zero-profit condition (10) yields

$$q_i = \sigma K^{\frac{\beta-\gamma}{1-\gamma}} r \quad \text{and} \quad q_{i^*} = \sigma K^{*\frac{\beta-\gamma}{1-\gamma}} r^*. \tag{12}$$

From demand functions (2) and (3), the world demand for variety i, d_i, is expressed as follows:

3) Rigidly, the agricultural good is produced in both countries if $\mu(\sigma-1)/2(\sigma-\mu)<1$ is satisfied.

$$d_i = c_i + \tau c_i^* = \mu p_i^{-\sigma} \left(G^{\sigma-1} I + \tau^{1-\sigma} G^{*\sigma-1} I^* \right). \tag{13}$$

Now, let us introduce parameter $\theta \equiv \{(\sigma - 1)\beta + \gamma\}/(1-\gamma)$ to express the degree of external economies of scale. Notice that $\partial\theta/\partial\beta > 0$ and $\partial\theta/\partial\gamma > 0$ hold. Using parameter θ and substituting equation (11) into equation (4), we have the price indices in the home and foreign countries as

$$G = \left(K^{1+\theta} + \tau^{1-\sigma} K^{*1+\theta}\right)^{\frac{1}{1-\sigma}}, \tag{14}$$

$$G^* = \left(\tau^{1-\sigma} K^{1+\theta} + K^{*1+\theta}\right)^{\frac{1}{1-\sigma}}. \tag{15}$$

Then, substituting equations (11), (14), and (15) into equation (13), the demands for varieties i and i^* are expressed by

$$d_i = \mu K^{\frac{\beta\sigma}{1-\gamma}} \left(\frac{I}{K^{1+\theta} + \tau^{1-\sigma} K^{*1+\theta}} + \frac{\tau^{1-\sigma} I^*}{\tau^{1-\sigma} K^{1+\theta} + K^{*1+\theta}} \right), \tag{16}$$

$$d_i^* = \mu K^{*\frac{\beta\sigma}{1-\gamma}} \left(\frac{\tau^{1-\sigma} I}{K^{1+\theta} + \tau^{1-\sigma} K^{*1+\theta}} + \frac{I^*}{\tau^{1-\sigma} K^{1+\theta} + K^{*1+\theta}} \right). \tag{17}$$

We now focus on consumers' income. From equation (10), the world capital income $rK+r^*K^*$ is equivalent to $1/\sigma$ of the value of the manufacturing aggregates. Since the total consumption of the manufacturing goods is given by $\mu(I+I^*)$, the following relation holds in the equilibrium:

$$rK + r^*K^* = \frac{\mu}{\sigma}(I + I^*). \tag{18}$$

Total income consists of labor income and capital income, *i.e.*, $I+I^*=L+L^*+rK+r^*K^*$. Then, using equation (18), we obtain

$$I + I^* = \frac{\sigma(L + L^*)}{\sigma - \mu}. \tag{19}$$

Substituting equation (19) back into equation (18), the total capital income becomes a function of the labor income:

$$rK + r^*K^* = \frac{\mu(L + L^*)}{\sigma - \mu}. \tag{20}$$

Let λ be the share of capital in the home country. Thus, $K=\lambda\bar{K}$ and $K^*=(1-\lambda)\bar{K}$. Here, we assume that the world capital endowment is owned equally by all capital owners along the lines of Baldwin et al. (2003). This implies that half of the capital employed in each country belongs to the home country's owners, regardless of λ. In view of this and using equation (20), the capital income in each country is expressed as $\mu(L+L^*)/2(\sigma-\mu)$. Therefore, we obtain the home and foreign incomes as follows:

$$I = L + \frac{\sigma(L+L^*)}{2(\sigma-\mu)} \quad \text{and} \quad I^* = L^* + \frac{\sigma(L+L^*)}{2(\sigma-\mu)}. \tag{21}$$

Since the two countries are symmetric, equation (21) shows that $I = I^* = \mu L/(\sigma-\mu)$.

Finally, we derive the equilibrium rental rates. From equations (12), (16), and (17), we obtain the equilibrium rental rates as

$$r = \frac{\mu\rho}{\sigma}\lambda^{\theta}\left(\frac{1}{\lambda^{1+\theta}+\tau^{1-\sigma}(1-\lambda)^{1+\theta}} + \frac{\tau^{1-\sigma}}{\tau^{1-\sigma}\lambda^{1+\theta}+(1-\lambda)^{1+\theta}}\right), \tag{22}$$

$$r^* = \frac{\mu\rho}{\sigma}(1-\lambda)^{\theta}\left(\frac{\tau^{1-\sigma}}{\lambda^{1+\theta}+\tau^{1-\sigma}(1-\lambda)^{1+\theta}} + \frac{1}{\tau^{1-\sigma}\lambda^{1+\theta}+(1-\lambda)^{1+\theta}}\right), \tag{23}$$

where $\rho \equiv I/\bar{K}$.

3.2 Forces at work

Now, let us clarify the agglomerative and dispersive forces in our model. We focus on two features of the model: external economies of scale and the market-crowding effect.

(i) External economies of scale

Because of external economies of scale, an increase in variety through capital inflow leads to declines in the marginal labor requirement, $B(n)$, and the fixed capital requirement, $F(n)$, in the manufacturing sector. Such declines affect the levels of the equilibrium rental rates derived in equations (22) and (23). From equation (10) and the market equilibrium condition $q_i = d_i$, we obtain

$$r = \frac{p_i d_i}{\sigma F(n)}. \tag{24}$$

Then, a decrease in $F(n)$ raises the rental rate, supposing that price and demand are fixed. Further, a decrease in $B(n)$ raises the numerator in equation (24) and thus raises the rental rate. From equation (9), a decrease in $B(n)$ lowers p_i. However, since the price elasticity of demand σ is greater than one, the decline in price increases the revenue $p_i d_i$.

In sum, the rental rate becomes higher in a country as more capital is employed (i.e., λ is greater) for the given price indices.[4] Therefore, the external economies of scale work as an agglomerative force.

(ii) Market-crowding effect

Since trade cost works as a trade barrier, competition is partly localized. When a firm moves from the foreign country to the home country through the movement of capital, competition in the home country becomes more severe, while competition in the foreign country becomes less severe. Then, capital flow from the foreign country to the home country shifts the home demand function for each variety downward through the increase in the number of firms in the home country.[5]

Without external economies of scale, such a downward-shift in demand lowers the relative rental rate.[6] Therefore, the market-crowding effect works as a dispersive force. Note that this effect declines as trade cost becomes lower.

4 Long-run Equilibrium

In this section, we analyze the long-run equilibrium entailing international capital movement. In particular, we focus on the effect of a reduction in trade cost on industrial location and welfare in the long-run equilibrium.

Capital moves to a country that offers a higher reward. With respect to the movement of capital, we consider the following adjustment process:

$$\dot{\lambda} = \Lambda(r - r^*), \tag{25}$$

with $\Lambda(0) = 0$ and $\Lambda'(\cdot) > 0$.

4.1 Industrial location

First, we focus on industrial location. Since the two countries are identical in all respects, the symmetric distribution of capital among countries always gives one of the long-run equilibria. In subsequent analyses, we assume that the economy is initially in this symmetric equilibrium. Then, if changes occur in the countries, for example, if unilateral FDI-attracting policies are implemented in one of the countries, then will this symmetric equilibrium remain stable?

4) The effect through the external economies of scale corresponds to λ^θ and $(1-\lambda)^\theta$ in equations (22) and (23).

5) This is also called the "local-competition effect." For details, see Baldwin et al. (2003).

6) This effect is expressed as τ in the numerator in equations (22) and (23).

Examining the stability of the equilibrium, we can conjecture about the pattern of industrial location.

Under adjustment process (25), we focus on the difference in rental rates between the home and foreign countries. Equations (22) and (23) derive

$$r - r^* = \frac{\mu\rho}{\sigma}\left(\frac{\lambda^{\theta} - \tau^{1-\sigma}(1-\lambda)^{\theta}}{\lambda^{1+\theta} + \tau^{1-\sigma}(1-\lambda)^{1+\theta}} + \frac{\tau^{1-\sigma}\lambda^{\theta} - (1-\lambda)^{\theta}}{\tau^{1-\sigma}\lambda^{1+\theta} + (1-\lambda)^{1+\theta}}\right). \tag{26}$$

From equation (26), we obtain the following result on the stability of the symmetric equilibrium.

Proposition 1

If the trade cost τ *is greater than the threshold value of the trade cost* $\bar{\tau}$*, then the symmetric equilibrium is stable; otherwise, the symmetric equilibrium is unstable where*

$$\bar{\tau} \equiv \left(\sqrt{1+\theta} - \sqrt{\theta}\right)^{\frac{2}{1-\sigma}}. \tag{27}$$

Proof. *See Appendix A.* ■

Proposition 1 states that the symmetric equilibrium may or may not be stable, depending on the level of trade cost. This implies that agglomeration occurs in the manufacturing sector as trade liberalization proceeds.

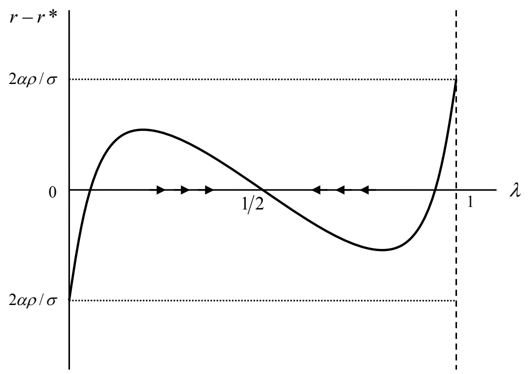

Figure 1 (a) : the case of $\tau > \bar{\tau}$

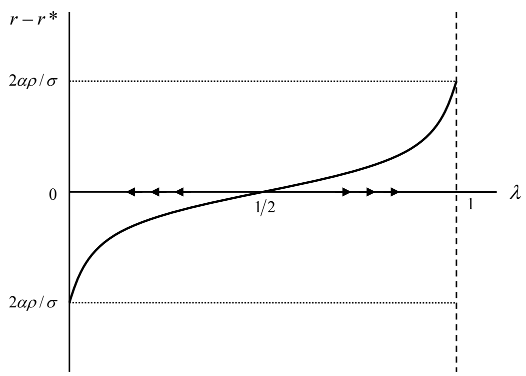

Figure 1 (b) : the case of $\tau \le \bar{\tau}$

Figure 1 depicts the values of the difference in the rental rate, $r-r^*$, for the share of capital in the home country, λ.[7] Figure 1 (a) is realized if the trade cost τ is higher than the threshold value $\bar{\tau}$, and the symmetric equilibrium (i.e., $\lambda=1/2$) is stable in this case. In contrast, Figure 1 (b) illustrates the situation where the trade cost τ is lower than $\bar{\tau}$, and the symmetric equilibrium is unstable. As we see from these figures, when τ is lower than $\bar{\tau}$, the manufacturing production is agglomerated to either of the countries; that is, all of the manufacturing productions are conducted in one country (i.e., an industrialized country) and the other country specializes in the agricultural production (i.e., an agricultural country).

Let us explain the reason for the result using the two features of the model stated in Section 3. Recall that the external economies of scale work as an agglomerative force, while the market-crowding effect works as a dispersive force. These two effects balance at $\tau = \bar{\tau}$. As the trade cost τ becomes lower, the market-crowding effect becomes smaller. If τ is less than $\bar{\tau}$, the agglomerative force via the external economies of scale outweighs the dispersive force via the market-crowding effect. Therefore, the symmetric equilibrium can be unstable.

4.2 Welfare

On the basis of industrial location, we examine the effects of the reduction in trade cost on welfare. In the following analysis, without loss of generality, we regard the home and foreign countries as the industrialized and agricultural countries, respectively.

The indirect utility function in the home country is given by $V=\mu^{\mu}(1-\mu)^{1-\mu}G^{-\mu}I$. As shown in equation (21), the home income is independent of λ and τ. Thus, it is sufficient to focus on the change in the price indices to clarify the effect of a decline in trade cost on welfare. Further, since we have $K=\lambda\bar{K}$ and $K^*=(1-\lambda)\bar{K}$, equations (14) and (15) are rewritten as

$$G = \left(\bar{K}^{1+\theta}\upsilon\right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad G^* = \left(\bar{K}^{1+\theta}\upsilon^*\right)^{\frac{1}{1-\sigma}}, \tag{28}$$

respectively, where

$$\upsilon \equiv \lambda^{1+\theta} + \tau^{1-\sigma}(1-\lambda)^{1+\theta} \quad \text{and} \tag{29}$$

$$\upsilon^* \equiv \tau^{1-\sigma}\lambda^{1+\theta} + (1-\lambda)^{1+\theta}. \tag{30}$$

From equation (28), an increase in υ leads to a decrease in G, and vice versa. The same mechanism also holds between υ^* and G^*. We thus use υ and υ^* as indices expressing utility

7) Figure 1 captures the global behavior of the function $r-r^*$, which can be derived in an algebraic manner. The details are available upon request to the authors.

in the home and foreign countries, respectively.

From equations (29) and (30), the values of v and v^* at the symmetric equilibrium (i.e., $\lambda=1/2$) are given by

$$v|_{\lambda=\frac{1}{2}} = v^*|_{\lambda=\frac{1}{2}} = \frac{1+\tau^{1-\sigma}}{2^{1+\theta}}, \tag{31}$$

and those at the agglomeration (i.e., $\lambda=1$) are given by

$$v|_{\lambda=1} = 1 \quad \text{and} \tag{32}$$

$$v^*|_{\lambda=1} = \tau^{1-\sigma}. \tag{33}$$

From (32) and (33), we find that welfare in the industrialized country is unambiguously higher than in the agricultural country. This is because the agricultural country must incur a trade cost for all manufacturing products, and purchase them at a higher price than the industrialized country. Moreover, comparing equation (31) with equations (32) and (33), we obtain the following results.

Proposition 2

(i) *Agglomeration unambiguously increases welfare in the industrialized country.*

(ii) *If $\theta < \bar{\theta}$ holds and τ is in the neighborhood of $\bar{\tau}$, then agglomeration decreases welfare in the agricultural country, while if $\theta \geqslant \bar{\theta}$ holds, then agglomeration increases welfare in the agricultural country, where $\bar{\theta}$ is the threshold degree of external economies of scale such that welfare in the agricultural country is unchanged before and after agglomeration.*

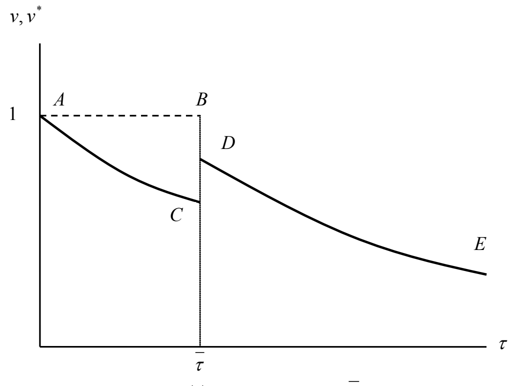

Figure 2 (a) : the case of $\theta < \bar{\theta}$

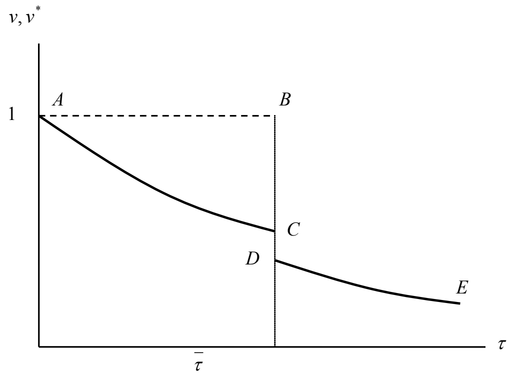

Figure 2 (b) : the case of $\bar{\theta} \le \theta$

Proof. *See Appendix B.* ■

Proposition 2 shows that external economies of scale are critical for the welfare effect of the reduction in trade cost. In particular, if external economies of scale are enough large, agglomeration makes both countries better off.

Figure 2 depicts the relationship between each country's utility level and trade cost. Figure 2 (a) illustrates the case for a small θ, while Figure 2 (b) illustrates the case for a large θ. Curves *DE*, *AB*, and *AC* express equations (31), (32), and (33), respectively. If $\tau > \bar{\tau}$, both countries' utility levels are shown by *DE*, and if $\tau \leqslant \bar{\tau}$, the industrialized home country's utility level is shown by *AB* and the agricultural foreign country's is shown by *AC*.

As the figure clearly indicates, the home utility level, and thus home welfare, is increased by agglomeration. Compared with the home country, foreign welfare is somewhat complex. In the case of Figure 2 (a), the foreign country may lower its own utility level by specializing in agricultural production. In contrast, in the case of Figure 2 (b), the foreign country raises its own utility level by specializing in agricultural production.

The intuition behind the results is as follows. Agglomeration of manufacturing production increases the number of varieties and lowers the price of manufacturing goods, which is welfare enhancing for both countries. However, for the agricultural country, agglomeration provides another effect: the burden of trade cost. This is disadvantageous to the agricultural country because it imports all manufacturing goods. Recall that θ is the parameter expressing the degree of external economies of scale, as stated in Section 3. If θ is small, the negative effect from the trade cost outweighs the positive effect from agglomeration, and vice versa. Therefore, if the degree of external economies of scale is lower (resp. higher), agglomeration causes foreign utility and welfare to deteriorate (resp. ameliorate). In contrast, the industrialized country enjoys the fruits of agglomeration.

5 Concluding Remarks

We have investigated the effect of a reduction in trade cost on industrial location and welfare in an economy with external economies of scale. We have proposed an analytically-solvable model concerning industrial location without losing accumulative agglomerative force, and have investigated the change in welfare when agglomeration occurs.

With respect to industrial location, we have shown that a reduction in trade cost is likely to lead to agglomeration. With respect to welfare, we have demonstrated that agglomeration makes a country unambiguously better off, whereas a country without agglomeration may become either better off or worse off depending on the degree of external economies of scale. This result implies that agglomeration may or may not be Pareto-improving.

In this paper, we have focused on industrial location and welfare in two symmetric countries. It is expected that some asymmetries, such as differences in production costs or degree of external economies of scale, will provide more realistic and interesting results. We would like to analyze these factors in our future research.

Appendix A: Proof of Proposition 1

Evaluating at the symmetric equilibrium, we find the value of τ such that the partial derivative of equation (26) with respect to λ is zero. Partially differentiating equation (26) with respect to λ, we obtain

$$\frac{\sigma}{\mu\rho}\frac{\partial(r-r^*)}{\partial\lambda} = \theta\left\{\frac{\lambda^{\theta-1}+\tau^{1-\sigma}(1-\lambda)^{\theta-1}}{\lambda^{1+\theta}+\tau^{1-\sigma}(1-\lambda)^{1+\theta}} + \frac{\tau^{1-\sigma}\lambda^{\theta-1}+(1-\lambda)^{\theta-1}}{\tau^{1-\sigma}\lambda^{1+\theta}+(1-\lambda)^{1+\theta}}\right\}$$
$$-(1+\theta)\left\{\left(\frac{\lambda^{\theta}-\tau^{1-\sigma}(1-\lambda)^{\theta}}{\lambda^{1+\theta}+\tau^{1-\sigma}(1-\lambda)^{1+\theta}}\right)^2 + \left(\frac{\tau^{1-\sigma}\lambda^{\theta}-(1-\lambda)^{\theta}}{\tau^{1-\sigma}\lambda^{1+\theta}+(1-\lambda)^{1+\theta}}\right)^2\right\}.$$

Evaluating it at the symmetric equilibrium, we obtain

$$\left.\frac{\sigma}{\mu\rho}\frac{\partial(r-r^*)}{\partial\lambda}\right|_{\lambda=\frac{1}{2}} = 4\left\{\theta-(1+\theta)\left(\frac{1-\tau^{1-\sigma}}{1+\tau^{1-\sigma}}\right)^2\right\}.$$

It follows that

$$\left.\frac{\sigma}{\mu\rho}\frac{\partial(r-r^*)}{\partial\lambda}\right|_{\lambda=\frac{1}{2}} \lesseqgtr 0 \quad\Leftrightarrow\quad \bar{\tau}\equiv\left(\sqrt{\theta}-\sqrt{1+\theta}\right)^{\frac{2}{1-\sigma}} \lesseqgtr \tau.$$

Then, if the trade cost τ goes below $\bar{\tau}$, the symmetric equilibrium becomes unstable. ■

Appendix B: Proof of Proposition 2

Statement (i) on home utility is straightforward from equations (31) and (32) because $\tau > 1$ and $\sigma > 1$.

We now prove statement (ii) on foreign utility. As shown in Proposition 1, agglomeration (resp. dispersion) occurs if $\tau \leqslant \bar{\tau}$ (resp. $\tau > \bar{\tau}$).

Let v_0^* and v_1^* be the foreign utility levels under dispersion and agglomeration, respectively. Note that v_0^* and v_1^* are given by equations (31) and (33). We consider the difference between these utilities, $v_0^* - v_1^*$. Define $\bar{\tau}$ as the trade cost level such that $v_0^* - v_1^* = 0$. Since $(v_0^* - v_1^*)|_{\tau=1} < 0$ and $\partial(v_0^* - v_1^*)/\partial\tau > 0$, we find that

$$\tau \lesseqqgtr \tilde{\tau} \quad \Leftrightarrow \quad v_0^* \lesseqqgtr v_1^*.$$

Whether agglomeration increases or decreases foreign utility levels depends on the levels of the trade costs τ and $\bar{\tau}$. That is,

$$\begin{aligned} &\text{Case (a):} \quad \text{For } \tilde{\tau} < \bar{\tau}, \quad v_0^* > v_1^* \quad \text{if} \quad \tilde{\tau} < \tau < \bar{\tau}, \\ &\qquad\qquad\qquad\qquad\quad v_0^* \leqslant v_1^* \quad \text{if} \quad \tau \leqslant \tilde{\tau} < \bar{\tau}, \quad \text{and} \\ &\text{Case (b):} \quad \text{For } \bar{\tau} \leqslant \tilde{\tau}, \quad v_0^* \leqslant v_1^*. \end{aligned}$$

We thus see that v_0^* is greater than v_1^* only if $\tilde{\tau}$ is less than $\bar{\tau}$ and τ is close to $\bar{\tau}$.

We then consider the relationship between τ and $\bar{\tau}$. Letting $\phi \equiv \tau^{1-\sigma}$,

$$\tilde{\tau} \lesseqqgtr \bar{\tau} \quad \Leftrightarrow \quad \tilde{\phi}(\theta) \gtreqqless \bar{\phi}(\theta), \tag{B1}$$

where $\tilde{\phi} \equiv \tilde{\tau}^{1-\sigma}$ and $\bar{\phi} \equiv \bar{\tau}^{1-\sigma}$. Equations (31) and (33) yield

$$\tilde{\phi}(\theta) = \frac{1}{2^{1+\theta} - 1}. \tag{B2}$$

Then, from equations (27) and (B2), we obtain

$$\tilde{\phi}(\theta) \gtreqqless \bar{\phi}(\theta) \quad \Leftrightarrow \quad \ln\left(\frac{1+\theta-\sqrt{\theta(1+\theta)}}{1+2\theta-2\sqrt{\theta(1+\theta)}}\right) \gtreqqless \theta \ln 2. \tag{B3}$$

Let us define the LHS in (B3) as $\psi(\theta)$, which is a strictly increasing and convex function satisfying $\lim_{\theta \to 0} \psi'(\theta) = \infty$ and $\lim_{\theta \to \infty} \psi'(\theta) = 0$. In this case, there exists a unique $\bar{\theta}$ satisfying (B3) with equality, and the following relationship holds:

$$\psi(\theta) \gtreqqless \theta \ln 2 \quad \Leftrightarrow \quad \bar{\theta} \gtreqqless \theta. \tag{B4}$$

From (B1), (B3), and (B4), we obtain

$$\tilde{\tau} \lesseqqgtr \bar{\tau} \quad \Leftrightarrow \quad \tilde{\phi}(\theta) \gtreqqless \bar{\phi}(\theta) \quad \Leftrightarrow \quad \bar{\theta} \gtreqqless \theta.$$

Therefore, Case (a) holds if $\theta < \bar{\theta}$ and Case (b) holds if $\bar{\theta} \leqslant \theta$. ■

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